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William Eckhardt

Paradoxes in Probability Theory

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William Eckhardt
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Gentle reader, I have sought to make this the finest, most balanced, most beautiful book possible, but could not subvert the law that like begets like.

Miguel de Cervantes, *Don Quixote*

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Introduction

The prevalence of paradoxes in probability theory can be judged against a subject famous for its paradoxes—mathematical logic, in which there are about half a dozen that are notable. (Russell's, Cantor's, Berry's, Burali-Forti's, Richard's, Grelling's and the Liar (Fraenkel et al. 1984) and (Heijenoort 1967)). Except for the Liar they all arose during the foundational period in the late nineteenth and early twentieth centuries. In probability theory (Szekely 1986) covers more than eighty paradoxes. (More advanced material of a paradoxical nature can be found in Romano and Siegel 1986). One can practically trace the whole history of probability theory in terms of paradoxes. And the probability paradoxes keep coming. Older paradoxes, settled and well understood, retain the capacity to instruct, astonish, and delight, but the paradoxes I have chosen remain controversial or are settled in a manner I believe to be incorrect.

We shall see time and again that the paradoxical nature of the materials drives analysts to resort to exotic, innovative, untested, fanciful, and otherwise dubious techniques. Proposed solutions are often as paradoxical as the original problems and only add to the mystification. The antidote is exclusive use of the near universally accepted elementary rules of probability theory. Five of these paradoxes have been widely discussed (Open Box and Card Experiment are new). Only two, the Betting Crowd and Two-Envelopes, are probability paradoxes in the narrowest sense. The others bear on questions of human nature, causality, human survival, cooperation, and controlling outcomes by actions. Each nevertheless hinges on the correct application of probability theory. The first three concern correct use of the concept of randomness, the second three the question of how probabilities should shape action. These problems are subject to definitive resolution in a manner that is routine in mathematics, but is in sharp contrast to the continual reassessment of old questions which is characteristic of philosophy. I claim that in every case correct application of probability concepts is the key to resolution of the paradox and that none of these applications are embroiled in foundational disputes. In other words, these are the kinds of probability problems whose solutions are independent of how one decides certain philosophical questions. The last claim rests

uncomfortably with the fact that much of the dispution on these paradoxes is found in philosophical publications. Paradox itself invites philosophical treatment; after all, the greatest paradoxes from Zeno to Russell have been philosophical. My contention is that patient application of near universally accepted probability concepts is enough to resolve each paradox. Probability theory gives definite answers to well posed questions; in particular probability theory cannot give inconsistent answers to an unambiguous question. Ultimately there are no probability paradoxes, only probability *fallacies*. There are profound philosophical problems associated with probability concepts: determinism, the open future, subjectivity, induction—all of these and more have been brought into disputations over these paradoxes. They just happen to be irrelevant to this set of problems. In briefest terms there are three issues:

(1) The validity of a certain kind of Anthropic reasoning (Doomsday, Betting Crowd, Simulation)
(2) The role of dominance principles in decision making (Newcomb, Open Box, Card Experiment)
(3) The treatment of divergent expected value expressions (Two-Envelopes).

The required applications of probability theory are unambiguous for three reasons: (i) only *discrete* probability theory is needed. There is no call for the continua of real analysis. (ii) these problems are finitary in nature; the exception is the Two-Envelopes paradox but even here a finitary version can be developed (9.6.3). (iii) the solution to these paradoxes are independent of foundational questions such as Bayesianism versus frequentism. Since the subjective probabilities in these paradoxes can be cast in terms knowledge of frequencies, Bayesian and frequentist demonstrations run in parallel. In finitary probability problems dependant on observable frequencies, there is little space in which ambiguity and equivocation can hide.

The last four paradoxes concern the optimality of strategies. Treatment of such problems needs principles of decision making to link probabilities to evaluations and actions. Its desirable that such principles be intuitively obvious, e.g., the player should prefer the larger monetary payoff to the smaller one. I wish to promote three such principles that I take to be incontestable.

(1) **The Symmetrical Ignorance Principle**: symmetrical ignorance forestalls rational preference.
Suppose everything a person knows about either of two options applies equally to both of them; then the person has no grounds on which to base a preference.
(2) **The Advisory Principle**: one should not recommend a strategy as optimal that fails to be optimal when the strategy in question is undertaken because of a recommendation.
There may be initial doubt as to whether such strategies exist, but there is no denying that if there are strategies that do not work when recommended,

then someone who wants what's best for the player should not recommend such strategies.

(3) **The Coherence Principle**: problems in outcome alignment should be played in the same way.
Suppose two games are played in a series of rounds. On any given round if the games are played the same way, they always yield the same payoff. This may be a different payoff than last time this strategy was tried, but on any round, the payoffs are the same if the strategy used in both games is the same. The coherence principle states that if one game has a best strategy, that strategy is also best in the other game.

It is difficult to see any reason to deny validity to (1) or (2) as decision principles; moreover, there seems to be no motive for contesting them. Matters are different with (3). It is as obvious and compeling as the other two, but if (3) is granted (with support from (2)) a solution to Newcomb's problem ineluctably follows that most experts consider completely wrong. On the conceptually treacherous terrain of Newcomb's problem, one may be reluctant to accept even the most transparently obvious assertion, at least until one can examine its consequences. The reader who has reservations about the status of (3) can read Chap. 4 as placing upon proponents of any other solution the added burden of explaining why two problems that have exactly the same responses to player strategies should be treated differently.

The short and infrequent bursts of mathematics in this book, mostly in the chapter on the Two-Envelopes paradox, are of the sort that some find elementary and others impenetrable. I have tried to make it possible to bypass the mathematical parts with minimal loss, but they are needed for a full understanding of Two-Envelopes and Newcomb's problem. I wanted the argument to be compelling enough for dissenters to feel the need to locate a specific mistake. Both rigor and transparency can compel, but they are somewhat at odds with one another. I can only hope I've struck a good balance.

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