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Causal Time Asymmetry[1]

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Abstract

Shunning passage of time explanations, the author seeks a reason for the earlier-to-later orientation of causality. The proposed formulation bases causal concepts directly upon those of coarse-grained entropy; in particular the time direction of causality aligns with the direction in which entropy increases. Further investigation shows that the number of possible causes for a given condition grossly exceeds the number of possible effects of the condition, consequently ruling out the possibility of necessary causes despite their frequent mention in the literature. The unruly diversity of possible causes means direct inference from present to past is nearly impossible. This leads to examination of how knowledge of the past is nevertheless possible and to a reinterpretation of the problem of induction. The author argues that at the ultimate level physical influence is bidirectional – coming from both past and future – but because of entropy increase, causality is detectable and exploitable in one direction only.

Keywords: causality, entropy, induction, retrodiction, time asymmetry

1. Introduction

This is an attempt to connect the temporal order of cause and effect to the time asymmetry of coarse-grained entropy increase and by implication to thermodynamic time asymmetry. We investigate not the origins[2] of entropic time asymmetry (henceforth t-asymmetry) but its consequences for causality. We make only one t-asymmetric assumption – that coarse-grained entropy increases from past to future – and seek to establish the connection of this empirically supported assumption to the asymmetry of cause and effect.

[1] I thank Robert Wald, as well as the referees, for substantial improvements in this article; the remaining defects are mine.

[2] The best candidates for the origin of t-asymmetry relate entropy increase to low entropy conditions near the Big Bang. The formulation herein proposed needs only the weaker assumption that entropy is *monotonic* – continually increasing or continually decreasing – for cosmological epochs; within an epoch "earlier/later" can be defined so that "later" points in the direction of entropy increase.

We mention two ways of accounting for the fact that entropy increase and causality point in the same direction.

i) The earlier-to-later direction of causality is absolute and inviolable; it is a logical concomitant of time and its passage. Time reversed causality is an absurdity. Entropy increases because low entropy states *cause* higher entropy states.

ii) The direction of causality derives from that of entropy increase; whatever "sets" the time direction of entropy increase thereby determines the direction of causality and of the experience of time's passage.

We follow the second route of basing causal asymmetry upon entropy increase. The fruits of this construction are several time asymmetric principles that go well beyond the temporal precedence of cause to effect, including a radical asymmetry in making inferences about what precedes a condition and what follows it. Unexpectedly it is making inferences about what precedes a condition that creates greater difficulties. We define *retrodiction* to be an inference about earlier conditions made *solely* on the basis of conditions at a later time; this makes it the exact time reverse of *prediction* which we take to be an inference about later conditions based solely on conditions at an earlier time. Our thesis is that inferences about the past are founded on prediction, not retrodiction.

Suppose we observe a wooden block resting on a table. We are powerless to retrodict how it got there: it might have fallen there or been tossed, knocked, placed, pushed, or lowered, and at what time we can hardly say. Given any scenario for which the block ends up on the table, one can concoct less likely causes with the same result, yielding an inexhaustible supply of alternative possible causes. In contrast we can make reliable *predictions* as to the results of dropping, placing or pushing the block on the table. The particulars of how the block hit the table now reside in unobservable microscopic distinctions in the noisy thermal background; to calculate this impact retrodictively we would have to reclaim all this noise. Unmanageable noise that makes no practical difference to prediction becomes a crucial factor in retrodiction.

The futility of retrodiction has been obscured by our greater knowledge of past than of future. Knowing more about the past than about the future, we presume prediction to be difficult and retrodiction easy; yet the opposite is true. The harmonizing factor is that prediction used to make inferences about the past is much more powerful than prediction directed at the future. Inferences about the past made by directly extrapolating back in time from current conditions lead to false conclusions such as that entropy was higher in the past. At the heart of the asymmetry of past and future inference is the fact that both are based on prediction, that is, on temporally forward reasoning, whereas t-symmetry would imply that past and future inference be the reverse of one another.

Improbability of cause and improbability of effect play highly dissimilar roles in causal analysis. An ordinary cause has only a miniscule probability of resulting in a bizarre effect, otherwise predictive inference would be worthless. A bizarre cause can have a high probability of resulting in an ordinary effect, hence retrodictive inference *is* worthless. Consider that horse and unicorn are equally good at leaving tracks on the beach. We rule out the unicorn not by retrodiction from the state of the tracks but by the near zero prior probability of a unicorn. Predictions most concordant with current conditions point to the likeliest prior conditions to have brought them about. In this way we can add to our store of prior conditions and probabilities.

The fount of all inference about the past is personal memory. Much of our confidence in memory is grounded in biology, reflecting the usefulness of such confidence to ancestral populations. For the question of warrantable inference, what is important is that memories can sometimes be checked. Since we cannot revisit the past to verify memories, the only remaining mode of confirmation is to compare current memories to objects, records and traces among current conditions. All confirmations of memory are confirmations of implied predictions: one predicts on the basis of memory that a certain record, trace or object will be in a certain condition; the prediction can then be confirmed or disconfirmed. This dependence on prediction is the characteristic of all inference about the past.

The great lost cause of empiricism – the problem of induction – can then be placed in a new light. Traditional treatments of induction interpose a conceptual chasm between past regularities and future regularities; this is to misjudge the nature of our knowledge of the past. Induction hinges on whether inferences about past regularities warrant predictions about future regularities. Since both rest on predictive inference, the inductive premise and conclusion are of equal status; they are both true, both doubtful, or both false. There is no inductive chasm between past and future.

Phase spaces such as those used in statistical mechanics are the best format for the investigation of entropy. We argue that, not coincidentally, they are also the best format for the treatment of causality. Phase spaces are abstract multidimensional arrays of all possible states of a system. Conditions and properties correspond to *regions* consisting of all states which possess the property or fulfill the condition. The development of these states in time defines a continuous flow. Causal relations among various states of a system can be treated in terms of geometric relations among regions of phase space. Whether one condition causes another is largely dependent on the degree to which the cause region flows into the effect region. Causal asymmetry is thereby tied to entropy increase. Because of turbulence in the phase flow a cause condition needs to be radically smaller than an effect condition for an appreciable amount of the former to flow into the latter, and because of this volume differential, innumerable small, low entropy, cause conditions can flow into a single large, high entropy effect condition.

I believe phase space also to be an appropriate framework for treating the perplexing questions of the influence of later events upon earlier ones. But first we turn to a dominating temporal conception that impedes research into causality and relegates to the nonsensical all attempts to assess reverse influence.

2. Time's Passage

There is an explanation of why cause precedes effect that is nearly universally accepted outside physics and philosophy – causality is earlier-to-later because time flows from earlier to later. The explanation is wholly inadequate. Consider the claim that future events cannot exert an influence on us because they do not yet exist. What is it about not yet existing that precludes exerting influence? It cannot be the mere fact of nonexistence, for other nonexistent events – past events – do exert such inference. It can only be that future events do not *yet* exist – they are non-existent and *future*; nonexistence does no work in the alleged explanation. Similarly the past is beyond control because it no longer exists, yet we can influence non-existent future events. It must be that past events are non-existent and *past*. It may add drama to causal asymmetry to cast it in terms of the past's instantaneous plunge into nothingness and future's unformed yet expectant nonexistence; however, what counts as evidence of the past's non-existence are variations on the theme that we cannot see, touch or act upon past situations; they are unalterable, and do not come around again. These address not the past's existence but its causal inaccessibility. In the following analysis we reason from entropy's increase to the past's causal inaccessibility, bypassing completely the ontology of other times as well as the circularities and question begging of passage-based explanations.

Consider the question of the interconnections of the quantitative[3] "arrows" of time, a quiver containing time asymmetries of thermodynamics, advanced and retarded radiation, quantum measurement, entropy increase, and cosmological expansion, to which I would add causal asymmetry. Which are fundamental and which are derivative? Assumption of time's passage undermines the question; the passage of time overrides any "arrow" of time by relegating both the past and the future to nothingness. No explanation of time asymmetry arises from this; the common direction of all time asymmetries, including the causal one, is imposed in a manner that forestalls further discussion. What asymmetry could stand up to the passage of time? How could advanced radiation move from the

[3] A second group that we call *qualitative asymmetries* are mostly the concern of philosophical investigators. This crowded quiver includes causal asymmetry, asymmetries of explanation and decision (Horwich, 8-10), closed past vs. open future (beginning with Aristotle) knowledge vs. intervention (Albert, 113-130), center vs. periphery (Popper), counterfactual dependence (Lewis), probalistic dependence (some interpreters of Bayes' theorem) and the fork and screening-off asymmetries (Reichenbach, 201-205). Let us say one time asymmetry *reduces* to another if the second can be used to account for or explain the first. The analysis herein proposed gives causal asymmetry a unique role in the network of reductions; each qualitative asymmetry is reducible to causal asymmetry and causal asymmetry can be reduced to the time asymmetry of entropy increase; this greatly narrows the field of candidates for irreducibility.

future that does not exist yet to the past that is already gone? Time's passage drags all temporal asymmetries into rigid alignment.

The elusive idea of time's passage is unique in that it gives the impression of providing answers to all questions of causal asymmetry while in fact providing none. The "destruction" of the past has been associated with causal asymmetry for millennia. It began as an evocative expression of the past's inaccessibility, analogous to the unreclaimability of something that has been crushed or immolated. It eventually came to be taken as the reason for inaccessibility. The idea has become so deeply ingrained that many theologians will not grant to Omnipotent Deity, with Whom they are customarily indulgent, the power of influencing the past. The extent to which our temporal conceptions are governed by compelling pseudo-explanations goes a long way toward explaining how time can be at once mysterious and transparent, the source of abiding perplexities and implacable certainties.

Whether time *truly* passes or not may be a question without a suitable answer. Whether to deny time's passage or to seek to circumvent it as explanatorily moribund may not be important. The problem is to trace the relationships of past, present, and future without appeal to this passage.

3. The Entropic Analysis

Theoretical physics is often considered not very relevant to the topic of causality; in fact there is a tradition in which progress in theoretical physics is seen as inimical to causality.[4] One reason is that analysis of causal relations is conducted mostly in terms of

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[4] A major motivation for this position is that comprehensive formulations from physics such as the Lagrangian or Hamiltonian display nothing that can be readily interpreted as causes or effects as philosophers understand the terms. This was certainly the case for Russell: "[causal laws] though useful in daily life and in the infancy of science, tend to be displaced by quite different laws as soon as science is successful… Certain differential equations can be found, which hold at every instant for every particle of the system…But there is nothing that could be properly called 'cause' and nothing that could be properly called 'effect' in such a system." (Russell, 153). (Cartwright, 12, 21, 74-75) portrays this conflict in terms of trusting law vs. trusting causes; compare a remark of Thom's "History gives another reason for the physicist's attitude toward the qualitative… Descartes with his vortices, his hooked atoms, and the like

qualitative narratives, e.g., oily rags in a hot, stuffy garage causing a fire. Theorists allow that physical causality depends on the laws of physics, but it is felt that the level of detailed physical process is the wrong one at which to analyze causal relations.[5] In any event disruptive tensions exist between t-asymmetric causality and t-symmetric physics.[6] There is a tendency to settle upon one of these as controlling, relegating the other to unimportance. In broad terms, one side rules out temporally reverse causation as illogical, inconceivable, and generally out of the question, thus disconnecting causality from time symmetric physical law; the other denies objective causal asymmetry – appearances to the contrary are attributed to subjectivity or anthropocentric illusion. What is needed to transcend these alternatives is a reconciliation of causal asymmetry and the time reversibility of physical law. An obvious strategy is to mimic the grand reconciliation of time asymmetry and reversibility, inaugurated by Boltzmann and completed by the Ehrenfests. I believe this path has been obstructed by misinterpretations that see causality as rock solid while reducing time asymmetry to the manipulation of subjectively chosen ensembles. This makes it seem a gross misconception to attempt to base the former on the latter. Entropy increase seems too flimsy to account for the irresistible march of causality.

In the *entropic analysis* we seek a quantitative account of causality in terms of the geometry of phase space, thereby making it possible to reconcile time symmetry with causal asymmetry. The analysis is built on three parallel contrasts – t-symmetric vs. t-asymmetric, fine-grained vs. coarse-grained, and concrete state vs. general condition. Some highlights:

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explained everything and calculated nothing; Newton, with the inverse square of gravitation, calculated everything and explained nothing." (Thom, 5). In the entropic analysis causality is characterized in terms of physical law in a Hamiltonian formulation.

[5] One exception is the *conserved quantity theory* (Dowe, 89-122) which connects causality to conservation laws; however this focuses upon a narrow aspect of the overall micro-process – conserved quantities – and conservation is t-symmetric ruling out an explanation of causal asymmetry along these lines.

[6] What is usually called *time symmetry* should more accurately be termed *CPT-symmetry*, i.e., so called charge and parity symmetries must be included in quantum field theory.

i) *Phase space and flow*. For a self-contained system, the collection of all possible states – called a phase space – has been an object of study by physicists and mathematicians for more than a century. The spaces that define the possible states of a macroscopic system are typically Euclidean spaces of many billions of dimensions. In this kind of phase space all microphysical degrees of freedom are assumed to be represented in a Hamiltonian formulation.[7] A single point refers to a possible state of the system. As the system develops in time, its phase point traces a trajectory in phase space. The trajectories of all points constitute the phase flow. Some adjustments are required for the quantum case, but in statistical physics there is less difference between classical and quantum versions than in other parts of physics (Tolman, 356-357) (Jancel, xxiv-xxvi). This owes to the probabilistic underpinnings of all statistical physics, which to some extent conceals the divergence between deterministic classical physics and indeterministic quantum physics. For simplicity I have relied on classical physics, an acceptable idealization in an investigation of macroscopic causality. All arguments can be adapted to the quantum case: individual systems are replaced by ensembles of identical systems and phase points by minimal regions; the treatments of coarse-graining and conditions remain essentially the same.

Phase flow is a mathematical transformation; it represents changes of groups of system states with respect to changes in time. It has nothing to do with the alleged "flow" of time. A *coarse-graining* is an exhaustive division of the phase space into finitely many regions that are disjoint except for shared boundaries. Coarse-graining involves the grouping of "similar" possibilities. We restrict attention to *observability coarse-grainings* for which this division has to do with what can be distinguished. Indistinguishable states belong to the same coarse-grained *cell*, a region of phase space. Distinguishability is judged relative to the *graining set*, a set of criteria and procedures for making physical distinctions. A typical graining set might include distinctions noticeable to a human observer and those resolvable by various measuring instruments

[7] Alternatively some degrees of freedom are suppressed, in which case there is contraction, not conservation, of phase fluid as kinetic energy vanishes into invisible motions (Abraham and Marsden, 187), or external influences can disrupt the purely Hamiltorian dynamics resulting in diffusion of the phase fluid (Mackey, 140-158).

and scientific techniques. In contrast *fine-grained* refers to the individual concrete state or history, including all microscopic detail at arbitrarily small scales. This is bedrock.

ii) *States and conditions*. We begin by respecting all objective physical difference no matter how slight, then introduce aggregation and equivalence. This leads to the pivotal distinction of *states*, *histories*, and *events*,[8] which are individual, and *conditions*, which are sets of possible states or histories – ensembles – hence general. (In order not to blur the contrast we take conditions as encompassing more than one state or history.) Momentary *states*, and lengthier *histories* are taken to be concrete individuals. In this formulation, as in statistical physics, *states* and *conditions* contrast in every way (the terms are unfortunately roughly synonymous in ordinary parlance). States are completely specified, individual, and momentary; conditions are general and loose enough to apply for longer periods. This contrast between the individual physical state or history and that which can be said to be shared by states or histories (including conditions, properties, descriptions, or membership in categories or ensembles) is paramount in the sequel. In phase space the contrast is explicit: states are points, and histories are *paths*, but conditions are *regions*. We take for granted the correspondence of system states to phase points and conditions to regions; we say “the system enters the condition” instead of “the phase trajectory representing the system enters the region representing the condition.” When we refer to *the behavior of a condition*, we mean the behavior of the phase fluid originally within the region. (Conditions themselves are static; a green apple may turn red, but the condition of being a green apple does not change.)

Causality is about making differences. There are always innumerable factors, including microscopic details, that count as not making a difference to the matter at hand. Whenever it is claimed that something does not matter or does not make a difference, there is a tacit appeal to conditions; in a fine-grained sense anything makes a difference

[8] What physicists call events most often are states or histories; alas, what probability theorists call events are inevitably conditions. (*Elementary events* in probability theory, such as throwing a six with a die, group countless alternative physical situations under one event.) In statistical physics where physics is wedded to probability, the state/condition distinction is paramount.

to anything else (in its light cone). The only option is to take states that differ in irrelevant respects as equivalent; that is, the only way to proceed is to form conditions.

We refer to concrete states or histories as *concreta.* Concreta can be taken as objectively real, but conditions are *sets of possibilities* and therefore creatures of the mind. This framework connects subjective to objective: an objective concretum can veridically belong to a subjectively delineated property or condition. Conditions, in this terminology, cannot be concrete, in fact *concrete* facts, properties or conditions are oxymora. The slightest physical alteration transforms a state or history into a different state or history, whereas, with conditions there are physical differences that do not result in a change of condition. Concreta are said to be *in* conditions, a felicitous linguistic concordance with members' being *in* sets. There are innumerable possible states or histories in any nonempty condition, and any state or history is in innumerable conditions. Discourse is rife with ellipses in which concreta stand for certain of their conditions. As knowers or agents we relate to concreta exclusively through their conditions. We observe or learn about concreta, but what we observe or learn concerning these are always conditions. Our hopes, though fulfilled through concrete occurrences, center only on conditions. Our actions create concrete histories but we can intend nothing more exacting than conditions. A description may pertain to a state or history, but the description never delineates more than a condition. A nonfiction narrative may concern a concrete history, but any narrative presents only a series of conditions. Beneath a certain scale a photograph or picture ceases to depict; it may portray a state, but it yields only a condition. In general a representation can refer to the concrete, but what it represents concerning the concrete is always a condition. In this rendering truth is perspectival, not because we are encased in our own subjectivity but because reality is too finely grained for linguistic truth. Fine grained "truths" or "facts"[9]-are indescribable; all questions of truth or factuality turn on describable conditions.

[9] Often seen as the pinnacle of objectivity, facts are contrasted at every turn with subjectivity, whether of fictions, impressions, opinions or values. Yet diverse facts pertain to a single state or history and diverse possible histories are consistent with a single fact. So called concrete facts are merely less general ones; this is true even of quite specific facts: suppose a box of gas contains precisely 10^{20} molecules. This fact is consistent with an unlimited number of alternate arrangements of these molecules and is hence a general fact.

iii) *Singular vs. general causality*. An enduring dispute concerning the nature of causality is whether it is singular or general. Does the causal connection reside in an individual instance or does the instance gain its causal status through being a particular case of a general rule? The property realist holds that a physical property is inherent in a particular instance; the property subjectivist that a property is a grouping of instances. Analogously the singularist takes causality as inherent in single instances of cause and effect, but in the entropic analysis causality is a relation between two groupings that turns on the behavior of the cause grouping with respect to the effect grouping. An instance of causality pertains to a pair of concreta, the cause and the effect, but observing a concretum amounts to placing it within a condition. Accordingly making an observation of a causal relation means determining a pair of conditions. Causality is observable or repeatable[10] only to the extent that it is based on conditions. This makes it futile to approach causality in singularist terms. In a particular instance, for concrete *a* to be the cause of concrete b, it is necessary, in addition to any direct physical relation between *a* and *b*, for there to be *conditions* A and B, containing *a* and *b* respectively, such that A causes B.

Membership in a classical ensemble (or a quantum mechanical mixture) has no effect on the development of an individual system. Suppose an egg is crushed by a heavy block of iron. (It is best to avoid agents so as not to become ensnared in the question of how *generalia* influence human behavior.) What property of the block crushes the egg? Is it the block's heaviness? Or is it the block's weighing more than 10 kg? Or perhaps the block's weighing 12.2 kg. crushes the egg? It is not a question of alternative causes; these are all descriptions of a single cause. The question of which property crushes the egg is misposed. Any of the aforementioned properties suffice to crush eggs generally. Which operated in this instance? Loosely speaking, it can be any of them, but in a deeper sense, it is none; in a particular instance heaviness can only operate by means of a

[10] Repeatability is a matter of categorization. In a fine-grained sense no states or events repeat; only conditions can repeat. A given state or history belongs to numerous conditions of varying degrees of repeatability.

specific weight and a specific weight can only become a factor through detailed atomic interactions. Any condition or property can operate only by means of the concrete and fine-grained. This leads to the seemingly paradoxical position that causality is a relation between conditions and that conditions exert no influence. The resolution is as follows: in a particular instance the cause concretely influences the effect, but such fine grained influence is insufficient to qualify the instance as causal; additionally a relation among conditions is needed. Causality depends crucially on regions even though regions exert no influence on their component points. Our linguistic habits foster all manner of expressions in which properties or conditions are said to exert influence; these are figures of speech – synecdoches or metonymies – in which the general condition is ascribed the causal powers of the individual state or history. As for causal asymmetry, a relation that pairs concreta is trivially invertible. The singularist in effect treats causality as a fine-grained relation between individuals; no time asymmetry can be wrenched from this.

iv) *Asymmetry trivialized*. An effect can be the cause of another effect, and a cause can be the effect of another cause; this gives the impression that causes and effects are alike except for the mere fact of temporal precedence. In fact since Hume, most formulations of causality have consisted of a set of t-symmetric relations between cause and effect to which a stipulation is adjoined that the cause is the one that happens first. This format greatly diminishes the role of asymmetry in causality. The entropic analysis in contrast makes t-asymmetry the foundation upon which causality is built.

v) *Entropy.* The twentieth century saw an explosion of entropy concepts. The one we mainly rely on is coarse-grained entropy: if K represents a set of background conditions, then relative to K the *entropy* of a system is $\log(\mu(G_i)/\mu(K))$, where μ measures volume, and G_i, the ith coarse-grained cell, is the one to which the system belongs. Without K we can only fall back upon the vanishingly small $\mu(G_i)/\mu(U)$ where U is the entire available phase space. The entropy of condition A is its average coarse-grained entropy, $\frac{\sum_i P(G_i|AK)\mu(G_i)}{\mu(K)}$ where AK is the conjunction or intersection of A and K. By 'entropy' unmodified, we always mean this kind of coarse-grained entropy.

The conclusion that t-asymmetry arises from subjectivity, whether through the 'gross nature of our observations' (Jancel, 172) through the mathematics of averaging (Wannier, 403), or as a projection of our internal temporal asymmetry (Price, 305, 321) would be warranted only if these subjective procedures lead to the conclusion of entropy increase irrespective of a system's behavior. Because of t-symmetry, there is a physically allowable entropy decreasing history for every physically allowable entropy increasing history. Entropy decrease should then be as detectable as entropy increase, yet the former is never observed. It is reckless to suppose such pervasive imbalance owes to a subjectively or conventionally imposed bias. Causal asymmetry can be observed only through coarse-graining which is a subjectively based procedure. It does not follow that causal asymmetry is itself subjective.

4. The Multiplicity of Possible Causes

We mostly operate with phase space volume instead of with entropy; volume is related to entropy and has the advantage that relative volumes are directly relatable to conditional probabilities. We focus on macroscopic, terrestrial examples but much of the treatment generalizes to larger scales and other locales. We first investigate causality under the artificial assumption of a perfectly isolated system; our conclusions transfer to the more realistic cases considered subsequently. In this idealization the influence of the rest of the Universe is limited to a static potential such as the Earth's gravitational field modeled as unchanging; this constancy assures the potential does not "perturb" the system.

$\overrightarrow{A}$ represents condition A under the transforming influence of the phase flow for a period Δt; $\overleftarrow{A}$ represents A under the reverse flow for a period Δt. The length of Δt varies with applications. A is *smaller* than B if $\mu(A) < \mu(B)$; A is *tighter* than B if $A \subset B$. *Size* refers to volume, not linear dimensions. A condition is *synchronically describable* or simply *describable,* if it possesses a finite description referring exclusively to a single time. *D* is the set of describable conditions which we also call *D*-conditions. Descriptive elaboration is limitless, but there are limits to observation. It is a natural assumption that a coarse-grained cell is *D*-condition, i.e., observability entails describability. For convenience we also stipulate the converse: all *D*-conditions are coarse-grained, thereby excluding descriptions of what is in principle unobservable via the graining set. Coarse-grained cells are then minimal *D*-regions corresponding to maximal allowable descriptions.

Describability places a limit on the geometric complexity of a condition. A time shift of a *D*-condition results in a condition too deformed and idiosyncratic to be describable. (The restriction to the synchronic assures that conditions needing diachronic descriptions such as $\vec{A}$ for describable A are not counted as describable.) Let A*, the *coarse covering* of a region A, be the union of all coarse-grained cells that intersect A. A* is the tightest *D*-condition that contains A. Let $\mu^*(A) = \mu(A^*)$ be the volume of A's coarse covering. For $B \in D$, $\mu^*(B) = \mu(B)$ (if x belongs to a cell that intersects B, then x is indistinguishable from some point in B and therefore fulfills B's description). A nonzero time shift in either direction of a non-empty *D*-condition results in an indescribable region that inevitably has more μ^* volume than the original condition. *D*-conditions are therefore μ^*-minima.[11] Conditions can contract, for instance, indescribable $\overleftarrow{A}$ contracts to describable A; in fact there is a contracting condition for every expanding one. But the phase flow is too irregular for a densely packed *D*-condition to contract; hence a condition that contracts is not a *D*-condition. A condition about to contract is as geometrically complex as one that has just expanded; descriptions are not sufficiently informative to delineate a region with innumerable idiosyncratic convolutions.

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[11] This is a form of the generalized $\bar{H}$-theorem (Jancel, 167-172). The role of macroscopic measurement is played by a *D*-condition on which a uniform distribution is defined. This also counts as a fine-grained density, so at this moment fine-and coarse-grained entropy are set equal. At any other time coarse-grained entropy is greater than the unvarying fine-grained entropy. This conclusion is time symmetric: the fluid which contracts to a given *D*-condition and that which expands out from it form indescribable conditions whose coarse coverings are both of much higher entropy that the *D*-condition itself.

Let U, the universal condition, consist of the entire available phase space. Call a condition that occupies a tiny fraction of U, *positive*, its compliment in U, *negative*. Since U encompasses all physical possibilities consistent with the postulated matter and energy of the system the overwhelming majority of U's volume consists of states of amorphous homogeneity. Structured, non-equilibrium conditions are all small enough to be positive. Descriptions that are grammatically negative are always subconditions of tacit or explicit positive background conditions. "Not depressing the brake pedal caused the car to crash." The presence of a car is a positive condition; not depressing its brake pedal – a subcondition – is also positive. As a negative condition "not the depressing of a brake pedal" includes virtually anything and has little to do with cars or brake pedals. All cause and effect conditions are assumed positive. This approach is not afflicted with the problem of negative causes, namely that, since absence is not concrete or particular, it cannot act as a cause. Because of background conditions, absence of a positive subcondition is presence of the opposite positive subcondition; either serves equally well as a cause.

A *D*-condition that holds in a system does so for a finite time period (it takes time for the phase point to move through a region), even a lengthy one. Under the action of the flow the fluid in a *D*-condition loses its synchronic describability immediately because there are always borderline situations, near the condition's boundaries that flow in and out in a highly irregular manner. Exponential divergence of trajectories in certain directions assures μ^* expansion, and the high dimensionality of the space makes it exceedingly difficult for increase in volume to be moderate. Consider a phase space with 10^{20} dimensions, a small number for a macroscopic system. There are accordingly 10^{20} linearly independent directions in which fluid can expand. Suppose a region in this space were to expand by one millionth of one percent. In three dimensional space this slight alteration would occasion a comparably trivial increase in volume of about three millionths of a percent, but in the phase space the alteration increases volume by a staggering factor of $10^{10^{11}}$. Why concern oneself with such a labile measure? Because relative volumes govern the conditional probabilities essential to causality. For instance, if condition B refers to a time Δt later than A and condition A is assigned a uniform probability distribution (i.e., a system in condition A is deemed equally likely to be anywhere within A) then, since the flow is μ-preserving, $P(B|A) = \mu(\vec{A}B)/\mu(A)$. Under these circumstances $P(B|A)$ can be no larger than $\mu(B)/\mu(A)$; in particular if B is quite small relative to A, then $P(B|A)$ is negligible. Diachronic conditional probabilities can be converted to synchronic ones through the appropriate time shifts; in this case $P(B|A) = P(B|\vec{A}) = P(\overleftarrow{B}|A)$.

In comparing sizes of conditions within phase spaces of exceedingly large dimensions, corresponding to macroscopic systems, “huge”, “colossal” or even “large” are euphemisms for conditions that are more than 10^{30} times larger than others under consideration. Estimates based on ideal gas models indicate that an expansion of hundreds of orders of magnitude in a fraction of a second are not unusual (Cercignani, 23-25). Similarly probabilities said to be “small” or “tiny” are often 10^{-30} or lower. Under these circumstances tendencies associated with this relative largeness or smallness become insurmountable; as a practical matter a probability of 10^{-30} is indistinguishable from impossibility.

The phase spaces employed in this analysis display all physical degrees of freedom, even thermodynamic ones. Although factors such as the solidity of objects induce correlations among the particles making up the objects, thermal motions and the like nonetheless induce close phase points to separate exponentially in certain directions. In a phase space for a system of macroscopic dimensions there is always enough chaotic directions to assure diffusion at the condition’s boundaries.

In the case of perfect isolation the Hamiltonian dynamics assure that phase volume is conserved (Liouville’s Theorem); condition expansion refers to an increase in the volume of the coarse covering. A positive *D*-condition expands in this sense both under the phase flow and the reverse phase flow (fig. 1); this effect is completely time symmetric.

But an individual phase point or quantum ensemble moves with overwhelming probability to increasingly larger coarse-grained cells in one t-direction, termed forward (fig. 2). In the reverse t-direction, this same phase point travels to increasingly *smaller* cells. The reconciliation of the t-symmetry of physical law and the t-asymmetry of physical behavior, first illustrated in the Ehrenfest urn model, is especially transparent in this framework. Since the symmetry relates ensembles to ensembles and the asymmetry relates individual systems to ensembles, they do not conflict.

We propose to base causal asymmetry upon entropy increase; to this end we make only one t-asymmetric assumption, a standard one about entropy behavior:

> (TA) Sufficiently isolated systems in our world tend overwhelmingly to have lower entropy pasts and higher entropy futures.

TA concerns the behavior of certain individual systems not of conditions. Assumption of TA does not compromise the t-symmetry of the phase flow; it alters how we judge the significance or relevance of certain elements of the flow. In phase space dynamics, there is a scenario of a smashed egg's reassembling for every one of an intact egg's smashing, yet TA rules out spontaneous assembly as a possible cause of an intact egg. Treatment of causal asymmetry logically requires some t-asymmetric principle beyond t-symmetric physics.[12] We believe TA represents a minimum in this regard.

"Striking a match *causes* it to light". But other factors – dryness of match head, availability of oxygen, the match's not having been burned already – are also needed for the match to light; in the causal literature these often unmentioned factors are referred to as *background conditions*. It is generally conceded that the division of cause from background conditions turns on context, e.g., if the presence of oxygen were in doubt we might say, "The oxygen caused the struck match to light." In the entropic formulation a cause condition should be a conjunction AK, thereby according equal status to the contextually important cause conditions A and the background conditions K.

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[12] The statements in the proof of any theorem derived from t-symmetric premises can be time reversed to yield a valid proof of the time reverse of the theorem. From t-symmetry comes only more t-symmetry.

As an illustrative cause condition take C: ‘a certain large rock is propelled against a certain fragile window’. Something presumably caused the rock to fly at the window, but this is not included in C. We examine the mundane assertion that the rock’s being propelled toward the window usually causes the window to shatter.

Any two instances of the rock flying toward the window can be continuously related by intermediate cases of the same condition; hence C is a single arcwise connected region of phase space encompassing the countless ways in which the rock can arc toward the window (fig. 3). Most of these lead to a broken window. A positive *describable* condition such as C spreads out, yet without external influence the phase fluid originally in C does not dissolve; $\mu(C) = \mu^*(C) = \mu(\vec{C}) < \mu^*(\vec{C})$. It is described as growing tendrils, stretching into filaments, becoming braided or sheeted. These are three-dimensional metaphors. Since the fluid spreads throughout a region many orders of magnitude greater than its original volume, it is subject to stretching in countless directions and concomitant compression in countless others. Additionally numerous chaotic degrees of freedom induce a continual churning action resulting in a filigreed pattern of unimaginable complexity. $\vec{C}$, greatly expanded and highly convoluted, retains C’s original volume but is spread throughout a region many times larger.[13]

An effect condition to consider is S, ‘the window has been shattered’. There are many ways to obtain S without propelling the rock at the window, but these do not issue from the cause condition C. Scenarios that begin in C spread throughout a region E, most of which is in S, since the window usually breaks. We take E, which is $(\vec{C})^*$, as the effect

[13] This spreading is assured if the system dynamics are *mixing* (Mackey, 106-107). We justify this assumption when we come to consider external influences. Stronger than ergodicity, mixing means that a condition, under the action of the flow for a sufficiently long time, comes to be spread fairly evenly over the entire space (Petersen, 57). The Kolmogorov-Arnold-Moser (KAM) theorem shows that for certain dynamical systems, chaotic phase flow can coexist with regions of regular circulation. However there is reason to believe that even under perfect isolation KAM type stability would not occur in a system with billions of billions of degrees of freedom (Coveney and Highfield, 284-285). In any event with so many degrees of freedom the external perturbations, always present in realistic cases, would disrupt the invariance of such regions, thereby assuring the dynamics are mixing.

condition, definable as 'results of propelling the rock against the window or results indistinguishable from the above'. (For simplicity we have chosen an example in which cause condition and effect condition are temporarily disjoint. Sustained causation would require a more complicated analysis, that would be nonetheless causally forward without simultaneous causation.)[14] E is a *D*-condition that contains $\vec{C}$, hence relative to C, E has colossal volume. Conservation of phase volume implies that only an exceedingly small fraction of E comes from C. There are many ways to end with broken glass and a rock in the debris, other than propelling the rock at the window; such scenarios are in E, but do not come from C. We refer to this as the *multiplicity of possible causes*; it stands in contrast to the paucity of possible effects. The question of the *number* of possible causes or effects is admittedly ill-defined; what rescues the thesis from meaninglessness is that the asserted quantitative divergences span numerous orders of magnitude. "Multiplicity" does not truly capture the radicalness of the imbalance. The window can have been broken by a vast assortment of objects in an unruly variety of ways; the telltale rock can have arrived in place by similarly diverse means. Many of these are no doubt fanciful, but they are relevant when addressing the question of *possible* causes for condition E; at an everyday level of description, their variety is inexhaustible whereas the number of possible effects is perhaps a dozen. We can augment the number of possible effects by employing more discriminating descriptive criteria, but according to these criteria the number of possible causes rises concomitantly. At a given level of discrimination there are always many more possible causes of a positive *D*-condition than there are possible effects of that same condition. This is not a trick of language or of perspective; because of the t-asymmetry of entropy increase and the turbulence of the phase flow, the earlier cause condition can only fill a negligible portion of the later effect condition. Diverse low entropy causes can result in the same high entropy effect. In briefest terms entropic t-asymmetry makes it necessary to consider lower entropy possible causes and higher

[14] Causal asymmetry has been falsely attacked in connection with alleged cases of simultaneous causation; these are either examples of noncausal logical sufficiency (that it was raining throughout England caused it to be raining in London) or are relativistically invalid (Eckhardt). Within the entropic approach, nontrivial simultaneous causation cannot be defined.

entropy possible effects.[15] Because of the size differential there are always many more of the former.

To qualify as a cause, the cause's presence must raise the probability of the effect, but there is no converse requirement that the presence of the effect raise the probability of a purported cause. For example, a chimpanzee riding a pogo stick on the beach raises the probability of finding an indentation in the wet sand. But only under highly unusual conditions does finding an indentation in the sand meaningfully increase the probability that a chimp on a pogo stick has been about.

We proceed to a general derivation. Let A be any positive *D*-condition on a system containing macroscopic amounts of matter. Let C(A), A's *possible cause region*, be $(\overleftarrow{A})^*$; let E(A), A's *possible effect region*, be $(\overrightarrow{A})^*$. Suppose $0 < p \leq 1$. A *D*-condition C_i is *p-cause* of A if C_i is a subset of C(A) the possible cause region, and C_i has probability p or higher of bringing about A. A *D*-condition E_j is a *p-effect* of A if E_j is a subset of E(A), the possible effect region, and A has probability p or higher of bringing about E_j.

Proposition. *For p not too close to zero, the p-causes of A grossly outnumber the p-effects of A.*

Proof. Let C_i and E_j be respectively a p-cause and a p-effect of A with, say $p \geq .01$. $\overleftarrow{A}$ is spread throughout C(A) in varying concentrations. Since $\mu(C(A))$ is enormous compared to $\mu(A)$, $\overleftarrow{A}$'s average density in C(A) is virtually zero. To be a p-cause of A, the density of $\overleftarrow{A}$ in C_i has to be at least p, making C_i a region of C(A) in which the concentration of $\overleftarrow{A}$ is well above average. This requirement greatly favors small C_i.

[15] This is not a restatement of the well-known fact that a system's entropy increases from the time of the possible causes to the time of the possible effects, a direct consequence of TA. The entropies of possible causes and possible effects are condition entropies, not system entropies. At a given moment a system has only one entropy (relative to a coarse-graining) but is in diverse conditions of differing condition entropies, any of which might enter into a statement of causality.

Unless C_i is a minimal *D*-condition it has at least one *D*-subcondition of density p or higher, but nearly always it has many such subconditions; these in turn have their own subconditions of density p or higher. (A proper subcondition is more specific than the condition to which it belongs; there are usually several cause conditions, more specific than the original, that have the same or greater reliability in bringing about a given effect.) Given a way of attaining condition A, there are always more improbable ways of accomplishing the same thing. This well of improbability is bottomless.

When we turn to p-effects of A, the picture is completely different. To be a p-effect of A, E_j has to contain at least a portion p of all of $\vec{A}$. This requirement greatly favors large E_j. Conditional on A, the sum of the probabilities of a set of disjoint p-effects cannot exceed one.[16] This means that the number of disjoint p-effects cannot exceed 1/p, a small number. In particular for $p > .5$ there can be no more than one p-effect. The handful of likely p-effects contrasts starkly with the cascades of increasingly unlikely p-causes. (As p approaches quite close to zero, the excess of possible causes over possible effects is redressed since large p-causes and small p-effects begin to count.) ,

The most unadorned example of the principle is approach to equilibrium in a gas. What is required is to shift the well-known facts of the matter into a causal idiom. *Any* non-equilibrium condition is a p-cause of equilibrium, but equilibrium is a p-cause only of more equilibrium (the probability of a spontaneous move away from equilibrium is well below p). There are countless non-equilibrium possible causes, only one possible effect – equilibrium.

We review some examples in which possible effects appear to outnumber causes. Consider the roll of a die. This is conventionally judged to have only six possible effects, and as possible causes there are surely many more than six ways to roll a die. We can

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[16] If the E_j's are disjoint, $\sum_j P(E_j|A) \leq 1$. The corresponding statement for disjoint p-causes, $\sum_i P(C_i|A) \leq 1$ is not very restrictive since, without further conditions, $P(C_i|A)$ is negligible for p-causes.

greatly proliferate the number of possible effects by also taking into consideration the resting position of the die on the playing surface (to some finite degree of accuracy). However, applying these refined standards of description to possible causes, we would find more possible starting positions for the toss. The combinatorial explosion of possible causes based on starting positions, number of bounces, amount of spin, height of toss, etc., keeps possible causes well ahead of possible effects.

That effects may branch out and multiply is a consequence of expansion from the original *D*-condition. In the broken window example an alarm may be triggered, or neighbors may call the police. The isolation conditions enforce a consistent perspective on the episode. If police, neighbors, phone systems, etc., conceptualized as surrounded by an isolating box, are part of the effect condition, they have to be part of the cause condition. This massively proliferates the number of possible causes.

Causality is often portrayed as one system acting on another; however for entropic analysis both systems have to be idealized as operating in one isolating box. If we take as cause condition a spaceship from Earth on its way to a previously unvisited planet, the possible effects of this single potentially tight cause condition are boundless. However since the effects concern the planet plus spaceship system, circumstances on the planet should be seen as part of the cause as well as part of the effect. The cause condition that summarizes our knowledge before arrival might be tight with respect to the ship but would necessarily be loose with respect to the planet, thereby greatly proliferating the number of possible outcomes. However, for any description of a possible episode on the planet, there exist many describable possibilities as to how ship *and* planet came to be such as to result in the episode; the multiplicity of possible causes remains in evidence.

(Mill, 285-289) argues that a "plurality of causes" sometimes hinders inferences about earlier times. In the entropic analysis this obstacle is seen as intractable and pervasive; nevertheless there are cases in which a cause seems stubbornly unique. But note that with improved technology comes improved capabilities to simulate and counterfeit. For a describable terrestrial effect one expects that, given sufficient motivation, time, and resources, technologies *could* be developed for bringing about this effect in a variety of dissimilar ways. Currently anything that sufficiently resembles a chicken egg has as necessary cause a bird. It seems reasonable that under the right conditions such an object could be synthesized or simulated in several ways without benefit of birds. It does not matter whether this ever happens; if it is physically possible under some circumstances, it is physically possible. (Conditional possibility changes with conditions, but the category of unconditional physical possibility is unchanging). For a given effect, narrowing possible causes down to a manageable variety cannot be accomplished without invoking prior conditions. In cases of seemingly unique causes, prior conditions weak enough to be considered background conditions suffice.

5. Necessity vs. Sufficiency

The idea that a cause is necessary and sufficient (NS) for its effect, dating back at least to Galileo, still attracts modern proponents (Ayer) (Nerlich) (Taylor).[17] However there is the following:

Proposition: If A and B are *D*-conditions, then A cannot be both necessary and sufficient for B, except for the trivial case in which A and B are one and the same condition holding at one time.

Proof: For definiteness suppose A is followed by B after a period Δt. If there is a point in $\vec{A}$ that falls outside of B, then A is not sufficient for B. If there is a point in B that is not in $\vec{A}$, then A is not necessary for B. If A is necessary and sufficient for B, then $\vec{A}$ = B. If $\Delta t > 0$, $\vec{A}$ = B implies that A and B cannot both be describable. ,

A describable cause condition can never be necessary and sufficient for a describable effect condition. For sufficiency of a cause we need to assure a chain of events and rule out extraneous disruptions, so a sufficient condition needs to be tight; for necessity of a

[17] The perfect logical symmetry of NS conditions has convinced some that causality is time symmetric (Nerlich) (Chisolm and Taylor).

cause we need to encompass all ways in which the effect condition can be brought about, so a necessary condition needs to be loose. A sufficient cause has to be small enough to flow into the effect condition; a necessary cause has to be large enough for the effect condition to reverse flow into it. These requirements conflict to the extent that they cannot both be fulfilled.

Attempts to revive necessary causes through fatally ambiguous expressions such as 'necessary in the circumstances' (Mackie (1974), 38), or 'within the totality of other conditions that occurred, but only those' (Taylor, 299) feed off the false idea that conditions are "out there" in the way concreta are. Conditions come from carving up the possibilities. An individual condition can be added to or removed from a description or another condition, but a condition cannot be added to or removed from a state or history. (When possible worlds theorists posit a world exactly like ours except for one factor, they attempt to graft a condition onto a concrete state or history; no wonder this procedure needs miracles.)

Causes become necessary only through stipulation. A gunshot wound has as necessary cause the shooting of a gun; if instead the resultant wound is described only in terms of contemporaneous conditions, there are possible causes other than the shooting of a gun. Examples of causes necessary for reasons of stipulation are plentiful: mosquito bite, ink stain, ravages of the storm, ground beef.

6. The Futility of Retrodiction

The asymmetry most important to our knowledge of the world is that of feasible prediction vs. impossibly difficult retrodiction. The futility of retrodiction arises from the fact that cause and effect differ so radically in volume that the former can only account for a miniscule fraction of the latter; to fill the effect condition requires numerous causal conditions. On the basis of an effect *alone* one cannot say much about the cause. But we do not base inferences about the past on current effects alone; we continually avail ourselves of knowledge of conditions and probabilities that pertain to times earlier than the effect (fig. 4). We can surround, as it were, a past condition by using information about times earlier than, later than, and contemporaneous to the past condition. Knowledge of prior conditions and probabilities allow us to grade the plausibility of alternative causes, then to use predictions to see which hypothetical causes best accord with current conditions. [18] Prediction is more powerful for probing the past than the future. Imagine how much more effectively we could predict the nearer term future if we knew many essential facts about the longer term future; we are in this position with respect to the past.

The futility of retrodiction, a consequence of assumption TA, is quite general, the only exceptions being rare cases in which constant entropy models can be employed such as in celestial mechanics. From a positive *D*-condition A, the phase fluid expands both toward the future and toward the past (fig. 5). In the future direction the vast majority of individual trajectories are entropy increasing; entropy decreasing trajectories are flukes of extremely small measure. For making inferences about A's likely aftermath, TA tells us that flukish entropy decreasing trajectories are to be ruled out, but these are of such small measure they have no discernable effect on probabilities.

Moving in the past direction from A the vast majority of trajectories are entropy increasing also – this is required by t-symmetry. However, describing these circumstances in conventional temporal language, we would say that the vast majority of the trajectories *decrease* in entropy as A is approached from the past.[19] Only a set of

[18] An allegedly cryptic remark of Gibbs' that has been variously interpreted (Albert, 85-86) (Costa de Beauregard, 132) (Landsberg, 135) is, I believe, about the obstacles to retrodiction: "while the probabilities of subsequent events may often be determined from the probabilities of prior events, it is rarely the case that probabilities of prior events can be determined from those of subsequent events, for we are rarely justified in excluding the consideration of the antecedent probability of the prior events."

[19] Since the great majority of trajectories leaving A are entropy increasing, and the great majority entering A are entropy decreasing, a slightly smaller great majority do both. Consider four kinds of entropy behavior of trajectories passing through A: (a) decreasing to A, then increasing; (b) increasing to A, then continuing to increase; (c) decreasing to A, then continuing to decrease; (d) increasing to A, then decreasing. Standard arguments (Jancel, 364-5) show that the overwhelming majority of these trajectories are of type (a); there are an equal number of (b) and (c) cases, but they're extremely rare compared to (a); (d) is extremely rare even compared to (b) or (c). (An indication of why this is true can be gotten from the fact that histories of type (a) have the highest average entropies and histories of type (d) the lowest.) Note that (a) and (d) are individually t-symmetric; (b) and (c) are t-symmetric when combined.

flukish trajectories of extremely small measure increase in entropy as they approach A. In making inferences about A's likely past, TA tells us to rule out entropy decreasing trajectories – virtually every trajectory – and that matters have to be reckoned strictly in terms of the entropy increasing flukes. The futility of retrodiction shows in bold relief: prediction from A can be constructed on the basis of the major movements of the phase flow; retrodiction from A amounts to anticipating flukes.

Without prior conditions as a guide it is impossible to wend one's way through the labyrinth of innumerable individually unlikely alternatives. Solely on the basis of condition A's holding we can often make good predictions concerning later conditions, but we can make no useful inferences as to earlier conditions.

7. The Entropy-Causality Principle

The reduction of causal asymmetry to entropy increase is contained in the following.

The Entropy-Causality Principle: The t-direction from cause to effect is necessarily the same as the t-direction of entropy increase.

Outline of proof. Let A and B be positive *D*-conditions with A a cause condition and B one of its effect conditions. We address the issue of whether B holds at $t + \Delta t$ given that A holds at t, and write "A", "B" for "A holds at t", "B holds at $t + \Delta t$", etc. We wish to assess $P(B|A)$. All accounts of causality share something in common: presence of the cause raises the probability of the effect. Theories may differ in the application – the necessitarian might require the probability to increase from zero to one – but none exclude it. To be detectable, however, a cause must raise the probability of the effect to an observable extent. This requires that knowledge of A be of some use in inferring what condition occurs after A; in particular A has to have a nonnegligible probability of engendering B.

We say a number is *incalculably small* if, except for the fact that it is nearly zero, there is no theoretical or practical way of assessing its value. Let $\hat{A}$ be the *acceptable subset* of A, acceptable that is under whatever t-asymmetry assumptions apply. Under TA certain points of A are unacceptable because they issue in trajectories that decrease or fluctuate in entropy. As is well known (Tolman, 148-154) (Davies, 65) the set of points leading to such aberrant trajectories is of extremely small measure in A; the difference of $\mu(\hat{A})$ from $\mu(A)$ is incalculably small. $P(B|A) = P(B|\hat{A}) = P(\hat{A}B)/P(\hat{A}) = \mu(\hat{A}B)/\mu(\hat{A})$ which is indistinguishable from $\mu(AB)/\mu(A)$. Replacing A with $\hat{A}$ has no effect on probability calculations.

Suppose the direction of causality and the direction of entropy increase were oppositely aligned, say cause is prior to effect, but all sufficiently isolated systems show higher entropy pasts and lower entropy futures – a time reversed version of TA. To be the cause of B, A has to have a nonnegligible probability of bringing about B in which case A needs to be decidedly smaller than B. Because of reversed TA, the only trajectories that should count in assessing $P(B|A)$ are those that *decrease* in entropy as they move from A to B; those for which entropy increases – nearly every trajectory leaving A – or for which entropy fluctuates are unacceptable. Under reversed TA, $\hat{A}$ is a set of such small measure in A that $\mu(\hat{A})/\mu(A)$ is incalculably small. In comparing $\mu(\hat{A}B)$ to $\mu(AB)$, we need to consider that AB, the set of trajectories in A at t and in B at t + Δt, might consist of an unusually high fraction of entropy decreasing trajectories. The fact that the effect condition B must be enormously larger than the cause condition A virtually rules out the possibility that B could receive from A more than an average share of entropy decreasing trajectories mixed among vastly more entropy increasing ones. In other words $\hat{A}B$ is an insignificant part of AB, and $\mu(\hat{A}B)$ is incalculably small relative to $\mu(AB)$. As above $P(B|A) = \mu(\hat{A}B)/\mu(\hat{A})$, but now numerator and denominator are incalculably small relative to $\mu(AB)$ and $\mu(A)$ respectively, making estimation of $P(B|A)$ completely intractable.

Cause and effect probabilities are not estimated by direct comparison of the volumes of phase space regions, but it is the existence of a few predominant effects that occur with fairly stable frequencies that underlies the estimation of these probabilities. Under reversed TA, a condition lacks predominant outcomes with stable frequencies. When condition A holds, which entropy decreasing path a system follows depends too sensitively on its precise location within A. Chaotic [20] dependence on initial location within Â frustrates attempts to estimate outcome probabilities from diverse instances of A. Thorough knowledge of condition A would not help, even in principle, in assessing B's likelihood, since almost all of A would be irrelevant. An earlier condition knowledge of which is useless in inferring what condition comes at a later time, fails to be a cause, contradicting the premise that A is B's cause.

Corollary: In an entropy increasing world reverse causality is effectively impossible.
Proof: Time reversing the outlined argument, one concludes that reverse causality in an entropy increasing system is as unlikely as forward causality in an entropy decreasing system. ,

8. Influencing the Past

To the preceding one might object that influence of future upon past is physically impossible, irrespective of entropy considerations, and hence the corollary is vacuously true. We argue that fine-grained influence of future upon past is continual and ubiquitous but that reverse influence lacks the focused relationship to conditions that would qualify it as reverse causality. This may clarify the meaning of the Entropy-Causality Principle and its corollary. However, the idea of reverse influence of the future upon the past clashes violently with idea of time's passage. We have renounced the passage of time as an explanatory principle, but this confers a considerable explanatory burden: matters

[20] Phase flow and anti-flow are equally chaotic, but the chaotic behavior of a condition (or equivalently of a large random sample of instances of the condition) is in much grater evidence in the t-direction of entropy *decrease* in which the divergence of nearby trajectories is "magnified" by the observability of progressively finer differences of state (from a smaller cell, it generally requires less change of fine-grained state to register a coarse-grained change).

customarily attributed to the passage of time have to be otherwise explained. We attempt this for influence and causality.

The idea that events at one time *change* events at another time should be abandoned as a metaphor of proven capacity to muddy the waters. What is demonstrably the case is that events at one time *constrain* events at another time; otherwise the succession of conditions would be completely haphazard. We propose that the best way to treat causality, as well as our relationship to past and future, is in terms of physical constraint. In what is perhaps the deepest expression of the t-symmetry of physical law, whether classical or quantum, the constraint at the ultimate fine-grained level of earlier states upon later ones and later states upon earlier ones is precisely equal. We designate the equal and opposite constraint[21] of states upon one another as *influence*. Influence as we use the term is concrete, singular, and fine-grained. We reserve *causality* for relations that are describable or detectable and hence that make essential use of conditions.

Fine-grained influence is t-symmetric in accordance with fine-grained t-symmetry; earlier and later states constrain one another equally although not necessarily totally. In this fine-grained sense, future events constrain present events to the same extent past events do. Matters are different, however, when seen in terms of conditions. Owing to rampant entropy increase in one time direction, only earlier-to-later constraint is coarse-grainable. Past conditions constrain present conditions to a much greater extent than do future conditions giving a sense of solidity to the constraining past and of emptiness to the non-constraining future. Present conditions constrain future conditions to a much greater

[21] How do events at one time constrain events at another time? Much of the answer points to the intermediate events; at least we do not suppose that constraint would occur without the presence of the intervening events. This connection of events at different times explains equally the constraint of later on earlier and earlier on later. In the Stuckelberg-Feynmann interpretation the forward temporal motion of particles can be identified with the backward temporal motion of the corresponding antiparticles (Scandon, 114). Temporal motion in either direction is a picturesque way of describing the connections among events; a rendering in which state *a* develops into state *b* and one in which a CPT-reversed version of *b* develops into a CPT-reversed version of *a* can refer to two systems, as preferred by the flow-of-time advocates, but they can also be distinct descriptions of the same history. Influence and reverse influence share one medium.

extent than they constrain past conditions, giving us partial control of the constrainable future that we are unable to exercise with respect to the unconstrainable past.

Human activity constrains future conditions. For reasons of entropy increase we cannot exercise similar constraint over past conditions. The logic of constraining events does not depend on changing events. If occurrant events form a timeless manifold, then one changes neither past nor future. This does not amount to fatalism since there is, from the vantage point of the present, a crucial difference between past and future. We can take part in making future events what they are; it is too late to participate in making past events what they are.

In a world where multiple factors vie for the designation "cause of effect X," the weak constraint that later events place upon X cannot compete with the substantially stronger constraint placed upon X by some set of earlier conditions. The latter always wins this tug-or-war, except at a fine-grained level inaccessible to observation or manipulation. Among *D*-conditions, the coarse constraint of earlier conditions upon later conditions is often significant – this is causality – but the coarse constraint of later conditions upon earlier ones is negligible. Detection of influence requires a certain kind of coarse-grained relation between cause and effect, and this can be secured only in the forward t-direction. There can be reverse influence but no reverse causality.[22] Reverse causality does not conflict with logic or ontology, but with raging entropy increase. Where then are the snows of yesteryear? Villon imagined they had been annihilated. But perhaps they are crisp and white as ever, merely causally inaccessible.

9. Capturing the Past

Our knowledge of the past so exceeds our knowledge of the future that many are convinced the future is unreal. Another result of this imbalance is that it conceals the

[22] There are exotic formulations such as the Wheeler-Feynmann absorber theory and its generalizations (Cramer) advanced action theories (Price), and at the fringes, retro-signaling and time travel, that involve detectable retro-causality. The entropic analysis herein presented rests on standard physics.

feeble nature of retrodictive inference. In this section we consider some examples of how knowledge of the past is gathered in light of the futility of retrodiction.

Prior to the advent of biotechnology, one made a solid inference from pregnancy to insemination. The source of this reliability was the contrapositive causal inference: absence of insemination results in absence of pregnancy. This is pure prediction, an instance of the requirement that useful inference about the past is founded on prediction; only idealized entropy-free inferences circumvent this.

Mythical stories of creation or origination adhere only to one branch of the procedure for valid inference about the past. Had the Serpent, after the expulsion from Eden, truly been cursed to crawl forever upon its belly, it would indeed account for the currently legless condition of snakes. What is lacking is not causal effectiveness but a meaningful prior probability based on other past conditions.

There is evidence that ancient peoples interpreted the fossils they chanced upon as the remains of what we would call mythological creatures; we can at least be confident they interpreted these objects differently than a paleontologist who interprets a given fossil against a matrix of collateral knowledge concerning the epochs in question. It is a better grasp of prior conditions and probabilities, supported by a greater understanding of temporally forward geologic and nuclear processes, and not the development of retrodictive capabilities lacking in the ancients, that permits sounder inferences concerning the origins of fossils. The paleontologist compares current conditions (fossils and other records) with *predictive* inferences about the disposition of strata, fossilization processes, decay rates, etc. Knowledge of prior conditions and prior probabilities can shape hypotheses about past events, and these can be compared, via prediction, to current conditions. The story is one of prediction. What surprises is that prediction in the context of making inferences about the past is radically more effective than prediction of the future.

Prediction is the hidden foundation of inference about historical records. Records are special kinds of effects. For R in the present to be a valid record of P in the past, P has to be a cause of R through some chain of events. If R lacks the right kind of causal relation to P, then R is not a true record as with a forgery. There is no good reason to count R as a record of P unless we can predict that P is the kind of thing that can under favorable circumstances cause R. For instance, the historian knows enough about conditions in the Roman Empire to predict Romans tended to leave records and stories concerning those they considered politically or militarily important. In particular, the semi-permanence of written records under favorable circumstances, as well as the high likelihood that a skilled copyist will produce a mostly accurate text, are predictions. The hypothesis that Julius Caesar existed leads to predictions much in accord with our current situation as far as historical records of Julius Caesar are concerned. The reliable inference that Julius Caesar existed is built on predictions. If challenged, one might support Caesar's historicity by pointing out that it is difficult to imagine how this configuration of evidence as to Caesar's existence can have all been falsified. What this means is that one cannot concoct realistic possible prior conditions that exclude Caesar's existence and from which one could plausibly *predict* the current condition of the evidence.

Why do we believe there were no flying cows last year? That there are none now is not a convincing reason – if there were flying cows last year, they could easily have all crashed by now. Instead we draw our conclusion from various predictions. If flying cows had existed in the historical past, one would predict humans would have noticed and left accounts; we find no such accounts. We predict that had there been such animals in prehistory we would find fossil evidence at least of precursor or collateral lines, which we do not. We can also make reliable aerodynamic predictions that such a creature is nearly impossible to construct. We know there were no flying cows last year, not by retrodicting from their current nonexistence but by prediction from the hypothesis of no earlier flying cows and from the predicted obstacles to designing such cows if none are around to reproduce. We rely on prediction to conclude there were no flying cows last year.

The cosmologist does not investigate the origins of our Universe by extrapolating back from its current state. Instead, on the basis of factors such as scientific context, simplicity, and tractability, plausible early conditions are joined to plausible models to obtain predictions in approximate agreement with current conditions. These factors play a role analogous to that of prior probabilities in ordinary inferences about the past. (I owe this example to Robert Wald.)

There remain rare instances in which entropy-altering processes can be successfully ignored such as the celestial mechanics of the Solar System. For this we can disregard a host of geologic and meterological processes and make successful extrapolations using a constant entropy model in which prediction and retrodiction are symmetric. However questions such as the solar system's origin cannot be addressed through retrodiction from its current condition, because in this case entropy-altering processes cannot be ignored. The only fallback is to proceed predictively through plausible initial conditions and rough predictive concordance with the current conditions.

In inferences about the past one shuns retrodiction in favor of predictive reasoning, supported by knowledge of *prior* conditions and *prior* probabilities. How then can one justify an initial inference about the past? The past's reconstruction needs an anti-foundationalist approach; otherwise where does one begin? Personal memory is a caprice without biological evolution. The remote times of biological evolution are inconceivable without the historical past. The historical past is unapproachable without personal memory. Furthermore, memory in general cannot be verified without records, and records in general cannot be checked without memory. These may be mutually reinforcing, but there is no starting point. No one has ever made an inference about past events without relevant supplementary knowledge about past conditions, supplied in part by memory; whereas inferences about future events typically proceed without additional relevant knowledge of future conditions.

10. Induction

The problem of induction concerns the justification of future *predictions* based on known past regularities. If inferences about earlier times are founded on prediction, the problem has been incorrectly posed. It narrows the gap between inferences about past and future that both are founded on prediction. This deprives the venerable problem of its customary launching platform: a contrast between the solidity of the evidence for past regularities and the logically unwarranted imputation of these regularities to the insubstantial future. Inferences concerning the past are laden with the same assumptions as are inferences concerning the future, and it is these assumptions about prediction that are supposed in need of justification. To the extent that we can trust the project of investigating past regularities, we can trust the project of predicting the future. If the inductive premise is reliable, then prediction is reliable, and the inductive conclusion is warranted. If the conclusion is unwarranted, then prediction is unwarranted, and the inductive premise is unreliable. Inferences concerning both past and future depend on the same principles of prediction. The premise and conclusion of the induction principle can be construed as both true, both dubious, or both false. It misrepresents the nature of our knowledge of the past to suppose the premise true and the conclusion false.

11. From Isolation to Openness

The crucial issue in passing to a realistic model is lack of perfect isolation. A system's *quasi-isolated* if it is materially isolated and has only energetic coupling to the environment. For a mole of gas to occupy one side of a box, without a partition, restricts each particle to half its possible range of positions, so it occupies about a $2^{-10^{23}}$ part of the gas's equilibrium condition. If, absurdly, the gas were in a fine-grained state poised to regress into this condition, influences as weak as distant stars would knock the regression off course. (This casts the cosmic background in the oxymoronic role of making retrodiction and retro-causality "more impossible." The question remains whether the cosmic background gently reinforces a raging local t-asymmetry, or whether the latter originates in the former.)

With solid objects on the scene, expansion in some phase space directions would be slower than with a gas. However, objects break, weather, and wear down; solids partake in the dissipation of heat and sound. If a block has been tossed on a table, the information concerning the block's motion is lost at the moment of impact, through the spread of shock and sound waves, which are degraded into heat. The fact that macroscopic motions continually transfer to invisible thermodynamic degrees of freedom is sufficient to power significant entropy increase.

A casualty of the loss of perfect isolation is Liouville's Theorem: background conditions randomize the development of a non-equilibrium system inducing both fine-grained and coarse-grained entropy increase (Blatt) (Morrison). Under the influence of external perturbations a filigreed pattern eventually dissolves. This serves to intensify the degree to which points of $\vec{A}$ get lost in the effect condition. If the external perturbations dominate, the phase flow is thoroughly randomized; however for gentler perturbations the system dynamics predominate with the result that filigreed patterns build up rapidly and dissolve slowly.

To model causality in the perfectly isolated case we relied on two assumptions: that the system dynamics were mixing and that time reversible dynamics produced time asymmetric entropy increase for individual systems. In the quasi-isolated case we can dispense with these assumptions. Fine-grained entropy increase, in so far as it affects coarse-grained entropy increase, accelerates the latter. External perturbations that are random relative to the system's state make the dynamics irregular enough to be mixing (Mackey, 148-151) which assures μ^* expansion. We can further conclude in this case that conditions expand according to μ as well as μ^*, eventually dissolving to fill the entire space (Cercignani, 24-25).

Consider a series of perturbations acting on the system during a period Δt; the reversed series is random relative to practically anything, but not to the fine-grained state of the system at the end of Δt. This asymmetry originates not in the system but in the perturbations; they ply their influence in one time direction only. A perturbation is an interaction that creates a correlation; it cannot be instantaneous. Nor is it t-symmetric: correlation *follows* interaction, and this remains true throughout a perturbation no matter how brief. The cosmological background randomizes the system in one temporal direction; in the other it de-randomizes the system. Any other conclusion requires anomalous correlations[23] – correlations, not associated with any interaction, that are reminiscent of Liebnitz's pre-established harmony in which *all* causation proceeds by anomalous correlations.

Several authors have pointed out that since coarse-graining is a t-symmetric operation, it cannot imply approach to equilibrium. Since a primary function of coarse-graining is to reveal t-asymmetry, it would be counterproductive if coarse-graining were itself anything but t-symmetric. Many derive approach to equilibrium from equally time symmetric assumptions such as contact with an equilibrium heat bath, repeated molecular chaos, or uniform probability of occupancy throughout a coarse-grained cell. No wonder statistical physics has been reproached for deriving so much conclusion from so little premise. These techniques, including coarse-graining, all require a time asymmetric shove to get the demonstration started. Lack of clarity on this point has fostered the impression that coarse-graining acts in some unclear manner as the *origin* of time asymmetry. Hence (Mackey, 109) criticizes coarse-graining for being time symmetric and so inadequate for deriving approach to equilibrium. (Blatt) and (Morrison) offer external perturbations as an alternative that avoids coarse-graining.

iii) *Opening the Box.* To this point we have employed isolating boxes in the analysis, but hardly anything actually takes place in boxes. The analysis shares this artificiality with statistical physics, in which isolation underlies theoretical treatments which can be

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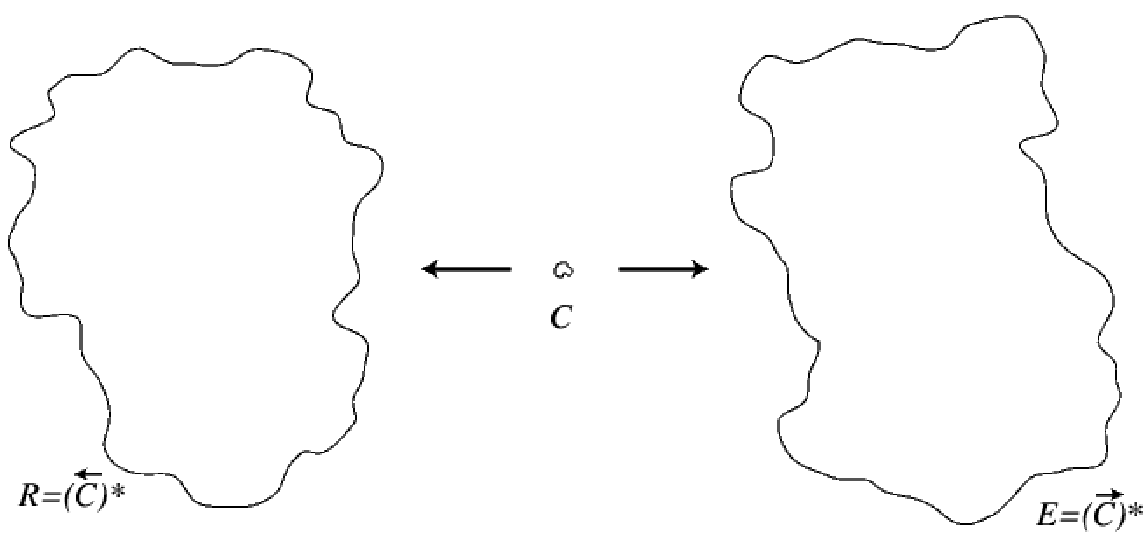
[23] If Σ is the entropy increasing background and σ is the entropy decreasing quasi-isolate (where earlier/later is defined from Σ's perspective) then relevant perturbations correlate with Σ, as the cause of the perturbations, and with σ, since the perturbations must correlate with the fine-grained details of σ so as not to disrupt the entropy regression. This set up requires correlations between Σ's initial conditions and σ's final conditions which do not arise from interaction.

empirically validated only under less exacting conditions. The question of what influences what, in a fine-grained sense, becomes hopelessly tangled in non-isolated cases; everything influences everything else at least a little. Causality is discernible because it turns on conditions comprising a substantial range of possibilities. This latitude renders some cases of causality robust enough to survive lack of isolation. In our terrestrial environment certain factors are seen to predominate in local episodes. This predominance is a matter of conditions: those factors can be ignored which have a weak enough combined influence so as not to alter the conditions by which the episode is characterized.

Gone is the possibility for rigorous phase space treatment, since the addition or subtraction of a single particle alters the dimension of a phase space thus forcing the reassessment of all metric and measure theoretic relations. The notion of fine-grained entropy is delicate and exacting; it cannot survive lack of material isolation. Conditions, more robust, can be insensitive to external circumstances. It is highly dubious that open systems possess an entropy in any sense. Openness presents a challenge to the concept of entropy mainly because openness presents a challenge to the concept of *system*; the term loses much of its power to designate objectively. The vagaries of what constitute the system undermine attempts to assign an entropy. In open systems the processes that underlie entropy increase in the quasi-isolated case – diffusion, transference of visible motions into invisible ones, degradation of chemical energy into heat, and t-asymmetric influence of the surroundings – are all in evidence. In quasi-isolated systems strengthening the degree of interaction with the background intensifies entropy increase. As the floodgates open further, can we doubt this intensification continues? Entropy increase becomes so virulent it destroys the very perspective by which we judge it.

We employ two kinds of schematics for representing phase space dynamics. In the first (figs. 1-3) cells or *D*-conditions are represented by flat regions and flow by arrows. In the second (figs. 4 and 5) cells or *D*-conditions are represented by darkened ovals and the phase flow by expanding conical shaded regions. The shaded region to the right of a condition represents the expansion of phase fluid from the condition; the region to the left, the contraction of phase fluid to the condition, or equivalently, the expansion from the condition under the anti-flow. In either representation a dashed path represents the development of an individual system.

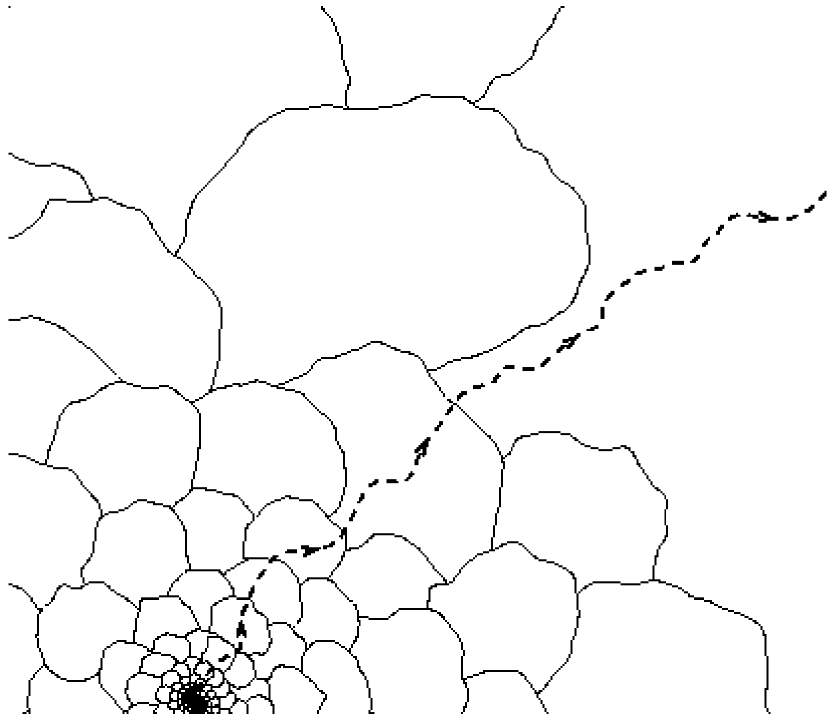
From condition C – a rock flying toward a window – there is symmetrical expansion in phase space both in moving forward to possible effects of C and in moving back to possible causes of C.



The part of R that goes to C represents the many ways the rock can come to be propelled toward the window. Only a small fraction of R can flow into the much smaller C; this reflects the fact that many *D*-subconditions of R have only a small probability of resulting in C.

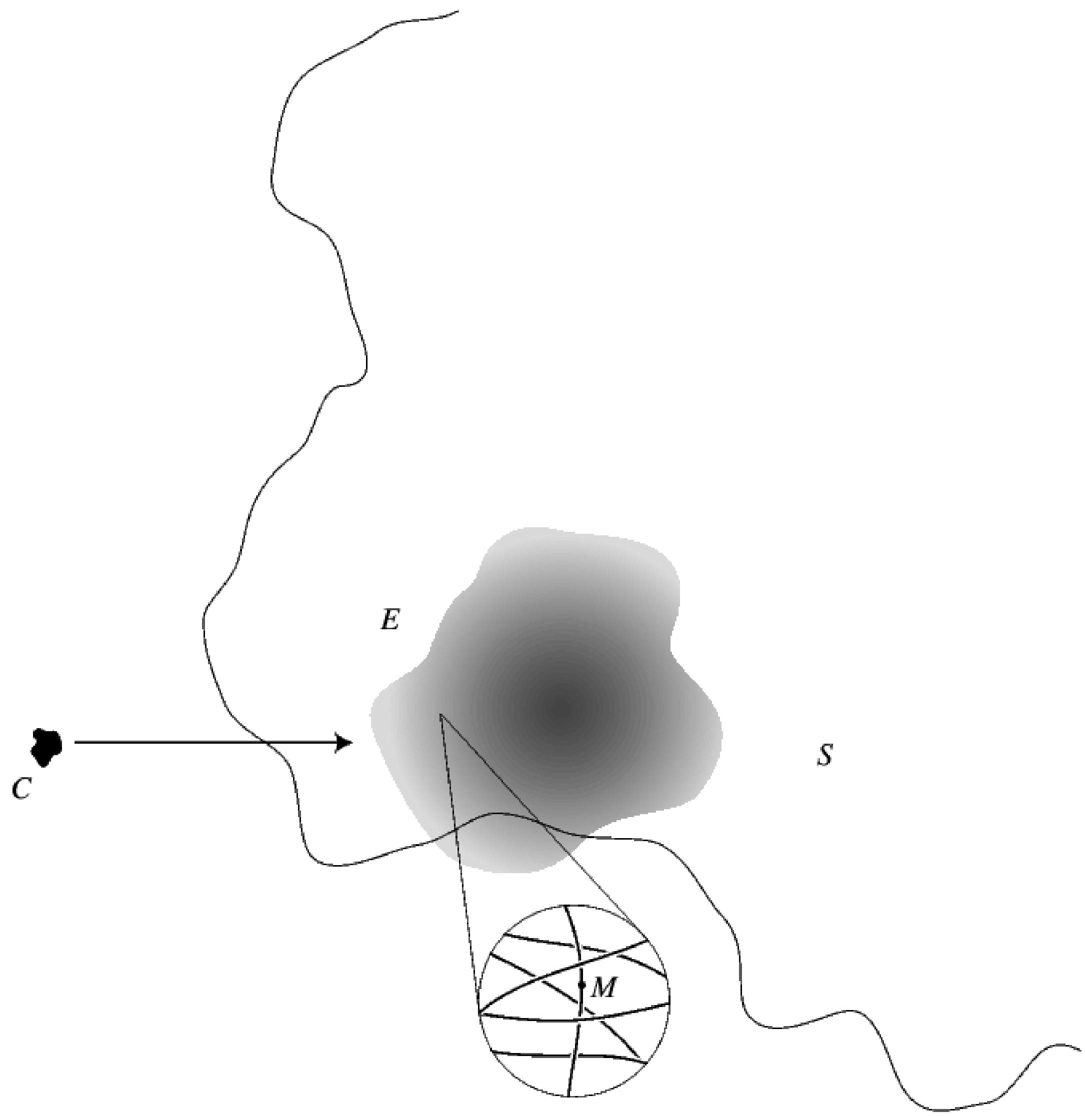
The part of E that comes from C represents the possible effects of propelling the rock at the window. This can be no more than a small fraction of E.

fig. 1



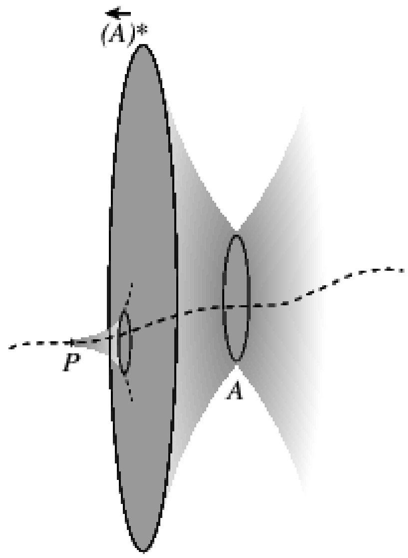
An individual phase point can behave t-asymmetrically by traveling through progressively larger regions. All or most of the fluid in a smaller region can flow into a larger region, but only the tiniest fraction of a larger region can flow into a smaller one. There results an *asymmetry of explanation*: a low entropy condition can account for a high entropy one but not vice versa. Moreover, for the case of entropy increasing toward the future, this means past conditions explain future ones but not vice versa.

fig. 2



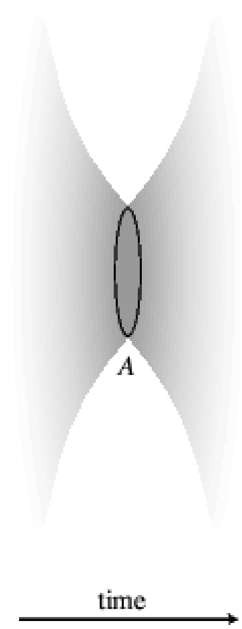
C is all ways the rock can fly at the window. E, the coarse covering of $\vec{C}$, contains the possible effects of C plus all states indistinguishable from these effects. A subregion of E that lies in $\vec{C}$, such as M has to be exceedingly narrow.

fig. 3



Only with a prior condition such as P can we narrow down the superabundant possible causes within $(\overleftarrow{A})^*$.

fig. 4



Because of the t-symmetry of the phase flow, the bundle of trajectories passing through *D*-condition A contracts to enter A then expands after leaving it.

fig. 5

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